Some puzzling problems in nonequilibrium field theories

Miguel A. Muñoz
Instituto de Física Teórica y Computacional Carlos I, Universidad de Granada, Facultad de Ciencias, 18071-Granada, Spain.
(February 1, 2008)

I review some open problems on the ever-growing field of non-equilibrium phase transitions, paying special attention to the formulation of such problems in terms of Langevin equations or, equivalently, field-theoretical descriptions, and their solution using renormalization group techniques.

I. INTRODUCTION

The introduction of the Renormalization Group (RG) ideas and their application to the study of critical phenomena constitutes one of the milestones in the spectacular development of the Statistical Physics during the last quarter of the twentieth century. The RG proved to be not only a powerful analytical tool to deal with complex problems (*i.e.* , problems with many different relevant scales), but also a conceptually beautiful and elegant theory, with a huge range of applicability. While RG ideas applied in (discrete) real space have helped to shed light on many problems [1], it has been in the framework of (continuous) field theoretical descriptions where, combined with perturbative methods, the RG has reached its most successful expressions. In particular, given a continuous field theoretical representation of a given statistical system at thermodynamical equilibrium, the identification of its critical points with fixed points of a conveniently defined RG transformation permits to obtain (perturbative) analytical expressions for the corresponding critical exponents. At the same time, by allowing to distinguish *relevant* from *irrelevant* ingredients in a rather systematic way, the combined use of field theories and the RG, has permitted to obtain elegant classifications of equilibrium critical phenomena and put under firm basis the concept of *universality* [2,1]. The representation of the Ising model universality class by the $\lambda\phi^4$ theory and its subsequent renormalization constitutes a paradigmatical instance [2].

Given this rather satisfactory scenario, theoreticians started wondering whether also critical phenomena occurring in systems away from equilibrium could be attacked using similar tools. In the lack of a well established theory for non-equilibrium phenomena, it is not straightforward to extend the equilibrium formalism to deal with non-equilibrium problems (for example, in these cases there is no partition function to be renormalized). The best way to do so turned out to be expressing such problems in terms of Langevin equations describing the underlying *dynamics* at a (continuous) coarse grained level. This procedure is valid not only to study general non-equilibrium processes but also relaxation to equilibrium states (the model A and B for the non-conserved and conserved relaxation dynamics of the Ising class are prototypical examples). In some cases Langevin equations representing given microscopic processes can be derived analytically using different techniques (among them: Fock space formalism combined with coherent state representations [3], and Poisson transformations [4]), while in many other cases they are just postulated from a phenomenological ground, by respecting what are considered *a priori* to be main symmetries, conservation laws, and other relevant dynamical constraints.

Experience teaches us that the richness and variety of phase transitions occurring away from equilibrium is by far much larger than that of equilibrium, and that in many cases it is very difficult to decide *a priori* what are the essential ingredients to be put into a sound Langevin description. Therefore, developing rigorous and systematic techniques envisaged to derive coarse grained Langevin equations from microscopic models is a high priority task within this context. On the lack of such general approaches one has to rely many times on phenomenological approaches.

Before proceeding, let us remark that any arbitrary Langevin equation can be written as an equivalent Fokker-Planck equation [5,4], and its solution expressed in terms of a generating functional (or equivalently and effective *action*) obtained as a path integral representation of the stochastic Langevin process. Therefore, in what follows "Langevin equations", "Fokker-Planck equations", "generating functionals", or "field theoretical actions" (Liouville operator) will be used interchangeably (see [6,2,7,3,4]).

In the forthcoming sections we report on a variety of interesting non-equilibrium systems, and present a *list of open problems* within this field.

II. THE DRIVEN LATTICE GAS (DLG)

The DLG is a variation of the kinetic Ising model with conserved dynamics, in which transitions in the direction (against the direction) of an externally applied field, $\vec{E}$, are favored (unfavored) [8,9,10], while transitions perpendicular to the field are unaffected by it. The external field induces two main non-equilibrium effects: (i) the presence of a net current of particles along its direction, and (ii) *strong* anisotropy. At high temperatures, the system is in a disordered phase while lowering the temperature there is (for half-filled lattices) a continuous transition into an ordered phase with high and low density aligned-with-the-field stripes. Elucidating the DLG critical properties is an important issue in the way to rationalize the behavior of non-equilibrium systems. The fol-

lowing Langevin equation was proposed some years back based on phenomenological arguments [11,10]:

$$\partial_t \phi(\mathbf{r},t) = \tau_\perp \nabla_\perp^2 \phi - \nabla_\perp^4 \phi + \frac{\lambda}{6}\nabla_\perp^2 \phi^3 + \tau_\parallel \nabla_\parallel^2 \phi - \alpha \nabla_\parallel \phi^2 + \eta(\mathbf{r},t), \quad (1)$$

where ϕ is the coarse grained field, η is a conserved Gaussian noise, and where the cubic term (a dangerously irrelevant variable [2]) is kept in order to ensure stability [11]. The fact that some of the predictions derived analytically from this equation (for instance, the order parameter critical exponent β takes a value 1/2) have not been convincingly verified numerically (a value $\beta \approx 0.33$ is systematically found in 2-dimensional Monte Carlo simulations of the DLG and variations of it [12,8,13]), has triggered further studies. These new analyses ended up with the proposal of a new Langevin equation aimed at describing the critical properties of the DLG:

$$\partial_t \phi(\mathbf{r},t) = \tau_\perp \nabla_\perp^2 \phi - \nabla_\perp^4 \phi + \frac{\lambda}{6}\nabla_\perp^2 \phi^3 + \tau_\parallel \nabla_\parallel^2 \phi + \eta. \quad (2)$$

This equation is a well known one: it coincides with the Langevin equation representing the random DLG (RDLG) [14] (for which the driving field takes values ∞ and $-\infty$ in a random unbiased fashion, generating anisotropy but not an overall current). This equation has been extensively studied in [14]; its critical dimension is $d_c = 3$ (instead, $d_c = 5$ for (1)) and its associated critical exponents and finite size scaling properties are now well known. At least two different recent numerical studies show that this equation reproduces rather nicely all DLG critical properties, and support the conclusion that it is the anisotropy and not the overall current the main ingredient characterizing the DLG phase transition.

However, the situation is far from satisfactory. The central issue is that naive power counting analysis shows that the current term establishing the difference between the two abovementioned theoretical descriptions is a relevant perturbation at the Eq.(2) RG fixed point and therefore, it is unclear why it should be absent in the proper Langevin description. It has been argued in [15,12] that the coefficient of this term happens to vanish in the limit $E \to \infty$. This would imply that the DLG has a sort of multicritical point in the infinite fast driving limit. This scenario still needs to be confirmed numerically [12,13].

Another theoretical way out of this puzzling situation is that the non-linear current term should be absent due to the fact that the microscopic theory is *fermionic* (i.e. occupation number restricted to be 0 or 1), while the Langevin equation describes, in principle, a bosonic process: the current term is required in this bosonic formalism in order to have a vanishing current in perfectly ordered aligned-with-the-field stripes (for which, the fermionic restriction precludes the presence of a non-vanishing current). We are presently working in the derivation of a field theoretical description that takes properly into account the fermionic nature of the microscopic model.

III. SYSTEMS WITH MANY ABSORBING STATES

Maybe the best well-known genuine non-equilibrium Langevin equation is the, so-called, *Reggeon* field theory [16]

$$\partial_t \rho(\mathbf{x},t) = \nabla \rho(\mathbf{x},t) + a\rho(\mathbf{x},t) - b\rho^2(\mathbf{x},t) + \sqrt{\rho}\eta(\mathbf{x},t) \quad (3)$$

that captures the critical properties of phase transitions into a single absorbing state (with no extra symmetries nor conservation laws), usually referred to as the directed percolation (DP) universality class [8,17,18]. The key property of Eq.(3) is that all terms (including the noise) vanish in the absence of activity, *i.e.* for $\rho(\mathbf{x}) = 0$). Even though convincing experimental realizations of this broad universality class are still missing, an overwhelming number of models have been studied, all of them sharing their critical behavior with this minimal Langevin equation. The situation is less satisfactory for systems with many different absorbing states [19], a prototype of which is the Pair Contact Process (PCP) [20,21]. In the PCP pairs of particles can generate new particles or get annihilated, but isolated particles do not have any dynamics; any configuration with just isolated particles is therefore absorbing. Models of this are relevant, for example, in catalysis [19]. The following Langevin equation for the PCP and related models was proposed some years back:

$$\partial_t \rho(\mathbf{x},t) = D\nabla^2 \rho(\mathbf{x},t) + a\rho(\mathbf{x},t) - b\rho^2(\mathbf{x},t) + \alpha\rho(\mathbf{x},t)e^{-w_1 \int_0^t \rho(\mathbf{x},s)ds} + \sqrt{\rho}\eta(\mathbf{x},t) \quad (4)$$

where the field ρ in Eq.(3) represents the density of pairs (activity), and the effect of the isolated particles (characterizing the different absorbing states [21]) is captured in the non-Markovian exponential term. It has been argued that the critical properties of this equation when approaching the critical point from the active phase are DP like [21,22]. Indeed, it is straightforward to see that in the presence of non-vanishing stationary activity the exponential term cancels out and we are left simply with DP. On the contrary, for spreading experiments for which critical propagation can occur inside the absorbing phase, the exponential term can be expanded in power series, and one ends up with [21]

$$\partial_t \rho(\mathbf{x},t) = D_2 \nabla_{\mathbf{x}}^2 \rho(\mathbf{x},t) + a\rho(\mathbf{x},t) + \alpha\rho(\mathbf{x},t) \int_0^t \rho(\mathbf{x},s)ds + \sqrt{\rho}\eta(\mathbf{x},t) \quad (5)$$

which is the well-known Langevin equation describing isotropic percolation dynamically *i.e. dynamical percolation* (DyP) [23]. These results are rather satisfactorily reproduced in numerical simulations [21]. Still there is a

point which remains obscure: If one works right at the critical point of the full theory, dynamical percolation terms are generated perturbatively, as first observed in [21] and recently stressed in [24]. This new vertex being more relevant than the dominant non-linearity in Eq. (3), leaded van Wijland to propose that the true asymptotic critical behavior should be controlled by a DyP fixed point. In order to generate an active phase (missing in DyP) he proposes to treat the term proportional to n^2 as a *dangerously irrelevant operator*, and finds an analytical expression for β. We believe that such a calculation cannot apply to the PCP since, even including the new term lacks of a well defined active phase. Being more precise, a term $-b\rho$ cannot compensate the linear in time divergence of $\alpha\rho(\mathbf{x},t)\int_0^t \rho(\mathbf{x},s)ds$ in the active phase.

Another open problem in this context is understanding within a systematic RG calculation how the background field (describing the different absorbing configurations) emerges as a *slave mode* of the activity field, *i.e.* how it inherits the critical properties of the order parameter in the active phase [22]. A comprehensive understanding of this family of phase transitions putting together the active and inactive phases is still missing.

IV. SELF-ORGANIZED CRITICALITY

The observation that *sandpiles*, the archetype of self-organized systems [25], fall into different absorbing states after every avalanche, right before new sand is added, opened the door to rationalize their critical properties using Langevin equations similar to those described in the preceeding section. The first step in order to do so was to regularize the sandpiles, by introducing the, so-called, fixed energy sandpiles (FES) which eliminate dissipation and addition of energy (sand-grains) [26]. This converts the total energy into a control parameter: large total energy generates stationary activity, while small amounts of energy lead the system with certainty to an absorbing configuration. The proposed set of equations for FES are:

$$\begin{aligned}\partial_t\rho(\mathbf{x},t) &= a\rho(\mathbf{x},t) - b\rho^2(\mathbf{x},t) + \nabla^2\rho(\mathbf{x},t)\\ &\quad + wE(\mathbf{x},t)\rho(\mathbf{x},t) + \sqrt{\rho}\eta(\mathbf{x},t)\\ \partial_t E(\mathbf{x},t) &= \lambda\nabla^2\rho(\mathbf{x},t)\end{aligned} \qquad (6)$$

where a, b, w, and λ are constants, and η is a Gaussian white noise. In these equations the activity dynamics is controlled by the same type of terms appearing in Eq.(3), plus an additional coupling between the activity field and a static *conserved* energy field. This extra term stems from the fact that creation of activity is locally fostered by the presence of a high background field density, and the energy is a conserved field, $E(\mathbf{x},t)$. The extra conservation law is therefore a new (relevant) ingredient with respect to RFT. Some other terms, consistent with the symmetries and conservation laws, could have been included in Eq.(6) but they all turn out to be irrelevant from a power counting analysis [26]. This same set of Langevin equations (plus higher order noise terms) has been derived using Fock-space techniques for other discrete models with many absorbing states and a static local conservation law [27]. The field theoretical analysis of this set of equations turns out to be a delicate issue (observe that analogous field theories for models with absorbing states but where the conserved field is not a static one can be studied perturbatively without any problem, [28]). As happens in the case of many absorbing states (without a conservation law) also here, at criticality DyP type of terms are generated. Here, even the physics coming from the active phase is not easy to work out. In this context, it has also been recently proposed [24] that the critical properties should be described by the "regularized" DyP fixed point, and again similar criticisms as those made before could apply here (although in this case, the problem is even more involved).

A successful RG calculation of this theory would be extremely valuable from a theoretical perspective; it would not only determine the critical exponents for a vast class of *self-organized* systems, but also clarify the issue of the proposed connection between self-organized criticality and the pinning of *interfaces in disordered media* [29].

Another related problem is that of the study of the effect of quenched disorder in systems with absorbing states [26]. A field theoretical analysis by Janssen [30], revealed the existence of running away RG trajectories, whose correspondence with the observed phenomenology in $d = 1$ and $d = 2$ [31] remains mysterious.

Before finishing this section, we want to point out that a promising formulation of the same problem, namely deriving an effective action for sandpiles has been recently addressed in [32].

V. OTHER REACTION DIFFUSION SYSTEMS

In this section we briefly enumerate some other open problems in field theoretical analyses of general reaction diffusion processes.

A. Two symmetric absorbing states

For some time it was believed that parity conservation (PC) was the main ingredient of a new, non DP, universality class [33]. By now, it is well established that the presence of an exact Z_2 symmetry between to equivalent absorbing states is its main distinctive trait [34]. Also, the introduction of parity conservation has been shown to play no relevant role in reaction-diffusion binary spreading models ([34], see also [35]). A field theoretical description of this universality class was proposed by Cardy and Täuber some years back. It starts with a Fock-space representation of the reaction diffusion lattice model in this class: $A + A \to 0$, $A \to (m+1)A$ (with

m an even constant) allowing to derive a field theoretical action. Even though the analysis of such a theory (that guarantees that the parity in the number of particles is conserved) is based on some uncontrolled expansion, it reproduces nicely many general features of this family, including the existence of a non-trivial critical point below two dimensions, the critical dimension $d_c = 2$, and some other properties. The formalism is also applicable to odd values of m where it also generates sound results [36].

An interesting open problem in this perspective would be to construct a more adequate field theoretical representation that should include as the main ingredient the presence of two symmetric Z_2 absorbing states, in the hope that in this more natural language a detailed standard RG procedure would be applicable.

Another related problem is that of writing down and analyzing a field theory for the Voter model [37] and extensions of it, for which a similar symmetry between different absorbing states appears (also the non-equilibrium kinetic Ising model at zero temperature belongs two this family of models with Z_2-symmetric absorbing states [33]).

B. Pair Contact Process with diffusion (PCPD)

In recent years, a new single-component absorbing-state universality class has been unveiled. It is the so called PCPD: if in the standard PCP we allow for diffusion of isolated particles we are led to this new class. Observe that switching-on diffusion represents a singular perturbation as, for instance, the many PCP absorbing configurations are reduced just to two, an empty one and one with a single wandering particle. It seems that the main ingredient in this class is the fact that reactions are binary (two particles are required for reactions to occur) and solitary particles travel performing random walks in between reaction zones. A field theory for this model was worked out by Howard and Täuber [38] some years back. Using a bosonic field theory (exact for a version of the model without a stationary active phase, usually called annihilation-fission process) they concluded that the critical dimension is $d_c = 2$ and that the transition is not DP-like. Unfortunately, the theory turned out to be non-renormalizable (i.e. an infinite hierarchy of relevant operators are generated perturbatively, making it un-tractable).

Different proposals have been made recently in order to rationalize the critical behavior of this class. These go from the existence of continuously variant exponents (as a function of the diffusion constant), to the existence of two universality classes (for small and large diffusion constants respectively), or just one well-defined set of critical exponents [39].

One possible strategy to analyze this problem from a field theoretical point of view is to introduce discrete models in this class with two different species: one corresponding to diffusing "isolated" particles, and one "diffusing-reacting" type of particle playing the role of the pairs in the original model [40,22]. This leads to a set of Langevin equations analogous to those proposed for the PCP, Eq.(4) but including diffusion of the secondary field. This changes the critical dimension from $d_c = 4$ to $d_c = 2$, but a systematic perturbative analysis allowing for a determination of the critical exponents has not been completed so far.

Similar problems are observed upon studying ternary-reactions (as $3A \rightarrow 0$, combined with $3A \rightarrow 3 + m$) for which new critical behavior is expected [41,35]. For higher order nth-reactions the upper critical dimension for annihilation is below $d = 1$ therefore no anomalous phase transition is expected to occur [36,35].

VI. DISCUSSION

We have briefly reviewed some non-equilibrium field theoretical open problems. They are related to non-equilibrium Ising-like models as well as systems with absorbing states. Other families of problems not discussed here are interfacial growth, non-equilibrium wetting phenomena, transitions described by the multiplicative noise equation as for example those occurring in the synchronization of coupled-map-lattices, etc. The existence of the various problems reported here gives raise to the following priorities for the developing of a systematic non-equilibrium field theoretical formalism:

i) the necessity of developing new tools for deriving Langevin equations (or field theories) in a systematic, rigorous way, from discrete microscopic models.

ii) Understanding the role of hard-core repulsion and/or implementing this constraint in a systematic way in field theoretical descriptions [42].

iii) Developing new analytical schemes, specially in low dimensions, to deal with problems for which standard epsilon-expansion does not yield satisfactory results.

It is my hope that this brief overview will stimulate further studies in this field.

It is a pleasure to acknowledge A. Achahbar, H. Chaté, C. da Silva Santos, R. Dickman, P. L. Garrido, G. Grinstein, R. Livi, J. Marro, R. Pastor-Satorras, M. A. Santos, Y. Tu, F. van Wijland, A. Vespignani, and S. Zapperi, for very enjoyable collaborations and/or enlightening discussions on the issues discussed in this paper. I acknowledge financial support from the European Network contract ERBFMRXCT980183, and the Spanish Ministerio de Ciencia y Tecnología (FEDER), under project BFM2001-2841.

[1] J.J. Binney, N.J. Dowrick, A.J. Fisher, and M.E.J. Newman, *The theory of Critical Phenomena*, Clarendon Press, Oxford, 1992.

[2] J. Zinn-Justin, *Quantum Field Theory and Critical Phenomena*, (Clarendon Press, Oxford, 1990). D. J. Amit, *Field Theory, the Renormalization Group and Critical Phenomena*, (World Scientific, Singapore, 1992).

[3] L. Peliti, J. Physique **46**, 1469 (1985). B. P. Lee and J. L. Cardy, J. Stat. Phys. **80**, 971 (1995). See also D. C. Mattis and M. L. Glasser, Rev. Mod. Phys. **70**, 979 (1998); and references therein.

[4] C. W. Gardiner, *Handbook of Stochastic Methods*, Springer-Verlag, Berlin and Heidelberg, 1985; and references therein.

[5] N.G. van Kampen, *Stochastic Processes in Physics and Chemistry* (North Holland, Amsterdam, 1981).

[6] R. Bausch, H. K. Janssen, and H. Wagner, Z. Phys. B **24**, 113 (1976). C. De Dominicis and L. Peliti, Phys. Rev. B **18**, 353 (1978).

[7] J. L. Cardy, *Scaling and Renormalization in Statistical Physics*, Cambridge University Press, 1996.

[8] J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models*, Cambridge University Press, (Cambridge, U.K., 1999).

[9] S. Katz, J.L. Lebowitz and H. Spohn, *Phys. Rev.* B **28**, 1655 (1983); *J. Stat. Phys.* **34**, 497 (1984).

[10] B. Schmittmann and R. K. P. Zia, *Statistical Mechanics of Driven Diffusive Systems*, in Phase Transitions and Critical Phenomena, edited by C. Domb and J. Lebowitz (Academic, London, 1995).

[11] H. K. Janssen and B. Schmittmann, *Z. Phys.* B **64**, 503 (1986). K.-t. Leung and J. L. Cardy, J. Stat. Phys. **44**, 567 (1986); ibid **45**, 1087 (Erratum) (1986).

[12] A. Achahbar, P. L. Garrido, J. Marro, and M. A. Muñoz, Phys. Rev. Lett. **87**, 195702 (2001).

[13] E. V. Albano and G. Saracco, Phys. Rev. Lett. **88**, 145701 (2002). See also S. Caracciolo et al. cond-mat/0106221 where a different conclusion is defended.

[14] B. Schmittmann and R.K.P. Zia, Phys. Rev. Lett. **66**, 357 (1991). B. Schmittmann, Europhys. Lett. **24**, 109 (1993). See also [10].

[15] P. L. Garrido, M. A. Muñoz, and F. de los Santos, Phys. Rev. E **61**, R4683 (2000).

[16] P. Grassberger, Z. Phys. B **47**, 365 (1982); H.K. Janssen, Z. Phys. B **42**, 151 (1981).

[17] G. Grinstein and M. A. Muñoz, *The Statistical Mechanics of Systems with Absorbing States* , in "Fourth Granada Lectures in Computational Physics", edited by P. L. Garrido and J. Marro, Lecture Notes in Physics, Vol. 493 (Springer, Berlin 1997), p. 223, and references therein.

[18] H. Hinrichsen, Adv. Phys. **49** 1, (2000).

[19] J. Kohler and D. ben-Avraham, J. Phys. A **24**, L621 (1991); D. ben-Avraham and J. Kohler, J. Stat. Phys. **65**, 839 (1992); I. Jensen, Int. J. Mod. Phys. B **8**, 3299 (1994).

[20] I. Jensen, Phys. Rev. Lett. **70**, 1465 (1993); I. Jensen and R. Dickman, Phys. Rev. E **48**, 1710 (1993).

[21] M. A. Muñoz, G. Grinstein, R. Dickman, and R. Livi, Phys. Rev. Lett. **76**, 451, (1996). M. A. Muñoz, G. Grinstein, and R. Dickman, J. Stat. Phys. **91**, 541(1998).

[22] M. A. Muñoz, C. A. da Silva Santos, and M. A. Santos, cond-mat/0202244.

[23] J. L. Cardy, J. Phys. A **16**, L709 (1983); J. L. Cardy and P. Grassberger, J. Phys. A **18**, L267 (1985). H. K. Janssen, Z. Phys. B **58**, 311 (1985).

[24] F. van Wijland, cond-mat/0209202.

[25] P. Bak, C. Tang and K. Wiesenfeld, Phys. Rev. Lett. **59**, 381 (1987); S. S. Manna, J. Phys. A **24**, L363 (1991). H. J. Jensen, *Self organized criticality*, (Cambridge Univ. Press, Cambridge, 1998).

[26] A. Vespignani, R. Dickman, M. A. Muñoz, and S. Zapperi, Phys. Rev. E **62**, 4564 (2000). R. Dickman, M. A. Muñoz, A. Vespignani, and S. Zapperi, Braz. J. of Physics **30**, 27 (2000). cond-mat/9910454.

[27] M. Rossi, R. Pastor-Satorras, and A. Vespignani, Phys. Rev. Lett. **85**, 1803 (2000). R. Pastor-Satorras and A. Vespignani, Phys. Rev. E **62**, 5875 (2000).

[28] F. van Wijland, K. Oerding, and H. J. Hilhorst, Physica A **251**, 179 (1998). See also R. Kree, B. Schaub, and B. Schmittmann, Phys. Rev. A **39**, 2214 (1989)

[29] M. Alava and M. A. Muñoz, Phys. Rev. E **65**, 026145 (2002).

[30] H. K. Janssen, Phys. Rev. E **55**, 6253 (1997).

[31] A. G. Moreira and R. Dickman, Phys. Rev. E **54**, R3090 (1996). R. Cafiero, A. Gabrielli, and M. A. Muñoz, Phys. Rev. E **57**, 5060 (1998).

[32] R. Dickman and R. Vidigal, J. Phys. A**35**, 7269 (2002).

[33] P. Grassberger, F. Krause, and T. von der Twer, J. Phys. A **17**, L105 (1984); H. Takayasu and A. Yu. Tretyakov, Phys. Rev. Lett. **68**, 3060 (1992); N. Menyhard and G. Ódor, J. Phys. A **29**, 7739 (1996); W. Hwang, S. Kwon, H. Park, and H. Park, Phys. Rev. E **57**, 6438 (1998).

[34] H. Hinrichsen, Phys. Rev. E **55**, 219 (1997).

[35] J. Kockelkoren and H. Chaté, cond-mat/0208497.

[36] J. L. Cardy and U. C. Täuber, Phys. Rev. Lett. **77**, 4780 (1997); J. Stat. Phys. **90**, 1 (1998). See also, U. C. Täuber, cond-mat/0205327.

[37] T. M. Ligget, *Interacting Particle Systems*, (Springer, New York, 1985). I. Dornic, H. Chaté, J. Chave, and H. Hinrichsen, Phys. Rev. Lett. **87**, 045701 (2001).

[38] M. J. Howard and U. C. Täuber, J. Phys. A **30**, 7721 (1997).

[39] E. Carlon, M. Henkel, and U. Schollwöck, Phys. Rev. E **63**, 036101 (2001). H. Hinrichsen, Phys. Rev. E **63**, 036102 (2001) and cond-mat/0208345. J. D. Noh and H. Park, cond-mat/0109516. K. Park and I. Kim, Phys. Rev. E **66**, 027106 (2002). G. Odor, cond-mat/0209287. R. Dickman and M. A. F. de Menezes, cond-mat/0227720; and references therein.

[40] H. Hinrichsen, Physica A **291**, 275 (2001)

[41] K. Park, H. Hinrichsen, and I. Kim, Phys. Rev. E **63**, 065103R (2001).

[42] One can incorporate the exclusion constraint in a bosonic theory: see F. van Wijland, Phys. Rev. E **63**, 022101 (2001). See also, S. Kwon, J. Lee, and H. Park, Phys. Rev. Lett. **85**, 1682 (2000); and V. Brunel, K. Oerding, and F. van Wijland, J. Phys. A **33**, 1085 (2002).